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How does a Carbon Tax affect Britain's Power Generation Composition?

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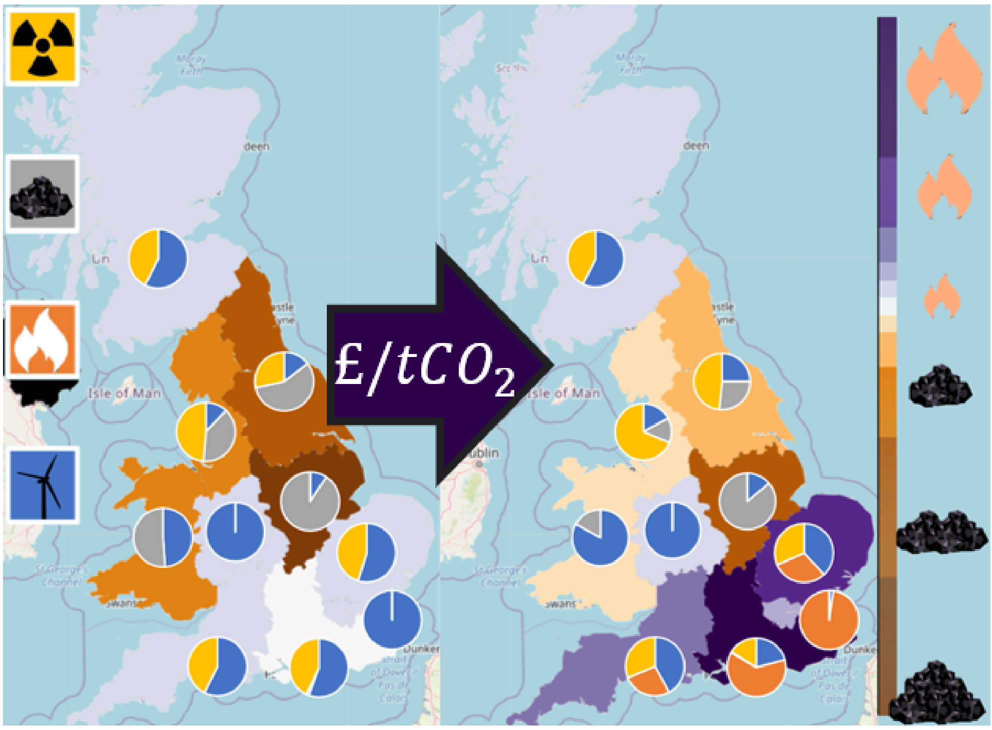
Abstract

The purpose of this paper is to determine the effect of different carbon tax rates on the power generation composition of Britain. This was accomplished via a regional model which considered both the differences in regional capacity by generator type as well as the high voltage transmission constraints and losses incurred by inter-regional power flows. This regional model is also compared to a pure dispatch aggregated model which considers only costs on the generator side (no transmission losses resulting from a regional breakdown) inclusive of the carbon tax. The effect of this tax is a transition from coal to combined cycle gas turbine (CCGT) generated power to fulfil demand unmet by nuclear or renewable sources. Due regional differences in demand and installed capacity technology types it is determined that more than 50% of this transition occurs prior to CCGT becoming more economical than coal from a pure dispatch perspective. Thus, due to CCGT generators typically being closer to larger southern loads than northern coal, transmission losses and the economic disincentive of a carbon tax combine in encouraging this transition.



Highlights

- OPF modelling performed at national and regional levels.
- Effect of different carbon tax rates on generator composition determined.
- Transmission losses accelerate coal to gas transition due to generator placement.
- Differences in regional generator compositions and loads found to be significant.

Contents

1 Introduction

Over the past few decades pollution reduction has grown in relevance as a global imperative. The goal of emissions reduction in modern times has resulted in numerous public policies seeking to respond to this objective by providing incentives to reduce more polluting technologies and incentivise those considered to be more environmentally sustainable. In the field of power generation a number of policies have been discussed seeking to address this policy direction. Examples of these policies include cap and trade schemes, carbon taxes and feed in tariffs as well as more targeted policies such as taxes on specific power sources (*e.g.* nuclear), taxes on more specific pollutants such as sulphur or nitrogen oxides, and renewable technology subsidies [4, 18, 31, 34]. By promoting efficient consumption and generation technologies (renewables, carbon capture, nuclear, *etc.*. .) energy policy seeks to improve sustainability [5, 7, 14–16, 21, 28]. In these cases policies are often analysed from a historic or, if modelled, from an aggregated perspective, factoring in economic dispatch and economic modelling particularly rather than specific grid conditions. Analysis of grid conditions, however, is not absent from economic policy studies, particularly in smaller scale analyses such as microgrid studies or other smaller scale resolution inquiries [9].

While some publications perform only analyses on aggregated data, in conducting an analysis cognisant of geographic and associated dispatch (high voltage transmission) constraints, a power system model was constructed. The modelling of power systems is commonly performed by an analysis of a system represented as a combination of elements and the connections between these elements. Whether it be smaller scale electrical systems, or multinational power grids some assumptions are required to reasonably reduce a system to such a representation such that it may be modelled. For a representative transmission network these elements take the form of busses, to which there may be associated loads or generators.

A 2018 dissertation from the Technical University of Munich makes such a model of the ASEAN power grid, analysing potential connections and generator compositions [17]. Furthermore a power grid model from Imperial College London divides up the island of Britain into a number of nodes with associated generators to approximate the UK's transmission system; analysing the effects of different wind power compositions [20]. Finally, a power-flow analysis for photovoltaic micro-generation is conducted in much higher resolution on Leicester city-centre, UK [23]. In all these cases can be observed that the analysis of power transmission systems requires modelling of a network of busses (or nodes) and branches (or connections).

The approximate number of busses and branches modelled also varies greatly in each case due to the trade-off between a model's scope and resolution. The Leicester model, for example, contains 3000 nodes, while the UK stability study contains fewer than 30 and the ASEAN analysis (while containing more nodes for other countries) simulates the grid of The Philippians (a nation of comparable size to the UK), with only 3. Thus resolution and order of magnitude of nodes may be seen to decrease with the scope and thus generalised nature of the model. In creating the model of Britain this document utilises, a network representation, though it may generalise network connections, must still seek to approximate the power network utilised in the UK, specifically focusing on high voltage (275 and 400 kV), long distance transmission [6]. The just cited Imperial College of Science, Technology and Medicine UK Electricity Networks report itself contains a particular example of power system simplification for modelling if the figures on pages 55 and 56 are contrasted, with the former figure from, The Electricity Association displaying a map of individual sites and overhead lines / underground cables, while the latter figure from the National Grid Company simplifies the generation/demand network of England and Wales into 6 regions (with additional connections to Scotland and France).

This paper performs an optimal power flow (OPF) study of a simplified representation of Britain's power network at different carbon tax rates to determine the effect this policy has on the UK's generation mix. OPF analysis is a well established power system modelling technique, with packages available in multiple programming languages [8, 13, 19, 29, 30]. By comparing the results of this analysis with a purely economic dispatch study the impact of grid conditions on the model's output may be ascertained while, within the bounds of the model's assumptions, the impact of different carbon tax rates on the United Kingdom's (UK's) power generation portfolio may also be determined.

Given that the UK already has a carbon tax, and that the inclusion of a carbon tax into an OPF model may simply be done via the cost functions of every generator, the consideration of a carbon tax policy was deemed most suitable for this investigation. Furthermore, as OPF modelling is concerned with the output of existing generators, this analysis will consider the present generator installations of the UK, rather than making additional provisioning for the direction of capital investment at different tax rates. As such the carbon tax's influence on the power source composition of the grid will be derived from its effect on the marginal cost of electricity production in specific.

The trade-off between resolution and scale is also an important topic of consideration. In a small scale analysis, power may be imported at substantial levels from external sources, while at a nation level, although most power is produced nationally, some energy may still be imported, but at a lower level, though the model itself must be more simplified than at a microgrid level. Finally, on a continental scale, the model will account for the transmission of power between nations, though this will be at the expense of resolution, greatly simplifying the transmission structure of these nations. For the purposes of this analysis, a national level is determined to be most suitable as it provides analysis on the same scale of the policy making institutions responsible for enacting environmental policies and by extension, it is also on the same scale as other national policy-based analyses. In summary, this paper will thus seek to analyse the impact of a carbon tax on the UK's power output portfolio as modelled by an OPF analysis of a simplified representation of the UK's power transmission network. For the UK, though the carbon tax is only part of government energy policy, the carbon tax (carbon price floor) was 16 GBP/tCO_2 in 2013 and is scheduled to increase to 70 GBP/tCO_2 in 2030 [1]. Though other sources may run on different presumed future rates, this paper will study a range of tax rates (until a higher tax rate incentivises no further change in generation composition) [10].

2 Model Formulation

In considering the optimal power generation combinations required by the United Kingdom, a model would be required to at a minimum consider the costs of each generation type. As such, a suitable solution to this necessity is to employ an economic dispatch model. Furthermore, due to the varied distribution of varying power sources throughout the UK, the national grid should also be considered. For these reasons the use of an OPF model (using the Newton-Raphson Method) was selected as the analysis technique for this investigation [2, 3].

To model Britain's grid data on generators, loads and the transmission system are required. The UK Government's Digest of UK Energy Statistics (DUKES) provides information on regional loads and generators over 30 MW in Britain [11, 12]. These same regions are used as given, with the exception of the smallest, the North East, which was combined with Yorkshire and the Humber for simplicity; resulting in a 10 region model. The full details on the model formulation methodology and resulting formatted data may be found in Appendix A, however, for simplicity, a summary will be described below.

With respect to loads, these are taken on a regional level. Generators are also allocated to the same 10 regions, with <30 MW generators (mainly small scale renewables) being dis-aggregated to a regional level from national capacity data in proportion to the allocation of known generators of the same type. The cost functions for these generators as well as their CO_2 outputs (to which the carbon tax is applied) are also determined by generator type, with costs being taken from the UK Department for Business, Energy and Industrial Strategy, while emissions levels are taken from the University of Victoria's Resource Economics and Policy Analysis Research Group (REPA) [10], [33]. Finally, the branches between these regions have their specifications taken from the averaged component ratings of overhead transmission lines.

3 Model Output

3.1 Composition

Using information from the noted government sources on generators, branches and cost, an OPF model constructed. In addition to this model's construction, to isolate the effect the grid parameters of the main OPF model a secondary model was made in which all loads and generators were placed onto a single bus; thus making it a purely economic dispatch model, in which where were no branch constraints, separated loads, or multiple generator locations effecting the output of the model. In both these cases the generic carbon outputs of each generator type were subject to a carbon tax, which was included in the cost function of each generator.

This carbon tax was increased from initially having no cost, up until 72 pounds per tonne of CO_2, by which point it was clear that the grid composition would not be further altered beyond that point as the least polluting sources were all utilised. As a generic cost function was used for all generators of the same type, an economic dispatch model would be expected to comprise of the cheapest generation source, followed by the second cheapest source once the first was fully utilised and so on. Without grid conditions this would specifically use power source types based on their generation costs as there will be no distribution losses or constraints. A typical hour's demand and renewable energy output is used, so although the analysis is for an instantaneous scenario, it applies to data corresponding to a typical hour.

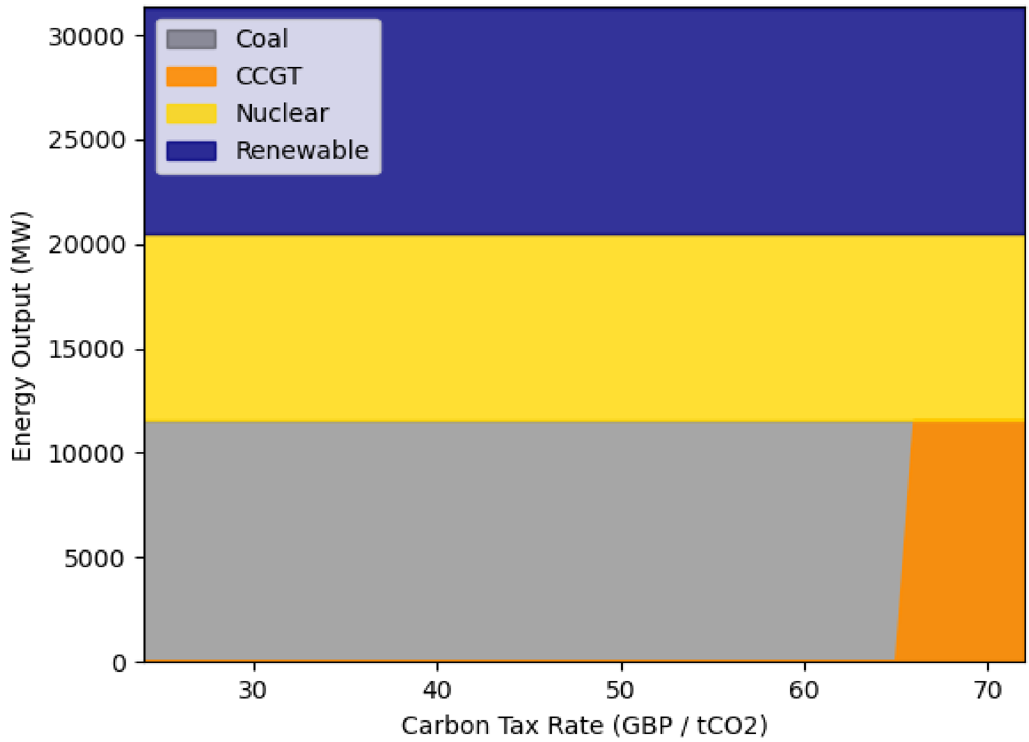


Figure 1: *Pure Dispatch Model (Displays the generator type composition in a 1-bus model at different carbon tax rates). Coal is transitioned to CCGT power at once between the tax rates of 65 and 66 GBP/tCO_2.*

As would be expected the perfectly inelastically modelled renewable output is always utilised to its maximum extent, followed by the cheapest power source as per the cost function data, nuclear energy. For both these generation types no emissions are outputted and thus no tax is applied for generation. Under the provided cost functions, which again should be noted were simplified to be the same for each generator type, coal power is initially the cheapest option. This, however, is replaced by combined cycle gas turbine (CCGT) at a tax rate of 65.2174 GBP/tCO_2.

With nuclear and renewable energy being fully utilised throughout, and given that CCGT has the lowest CO_2 output per MWh of every generation type remaining, once CCGT is fully utilised to fulfil the remaining demand not fulfilled by renewable or nuclear power then no further increase in the carbon tax beyond that tax will make any further change to the power output composition which can most affordably fulfil demand. The turning point from 65 to 66 GBP/tCO_2 will also be important to note in comparing these results with those from the 10 bus model.

Also, while grid constraints will also effect the 10 bus model, given a high enough carbon tax rate, so long as grid constraints (such as line capacities or exceptional losses) do no impose hard limits on the model which would outweigh a carbon tax of any size, it is to be expected that the same final composition is arrived at. This composition would be comprising of renewable energy, fully utilised nuclear power, and CCGT fulfilling demand not met by the first two source types. Furthermore, despite more power likely being required to accommodate for transmission losses, CCGT power is still far from fully utilised in the first case, so it is still expected that any demand unmet by renewable or nuclear power could still be met by CCGT.

Although the 10 bus model is expected to have a similar portfolio at a very low or very high carbon tax rate, at around the turning point from coal to CCGT observed in the 1 bus model, grid conditions are likely to have an impact on this transition. An important point of interest in this investigation is to note the extend and specifics of this transition. The output of the 10 bus model can be seen in Figure 2.

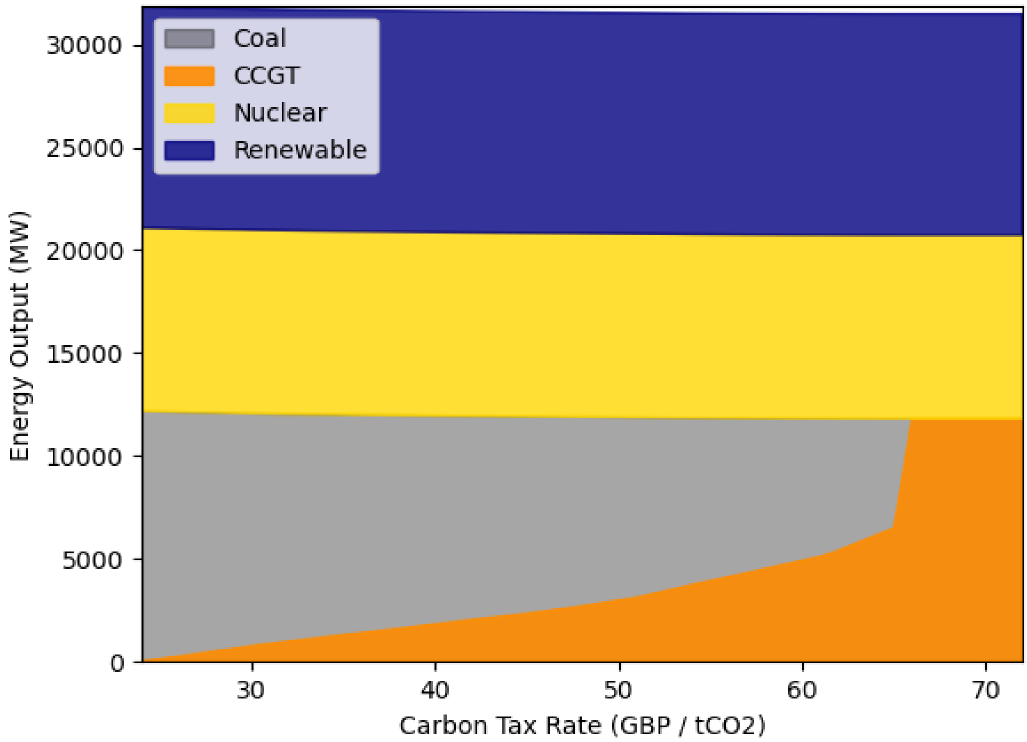


Figure 2: *OPF Model (Displays the generator type composition in a 10-bus model at different carbon tax rates). Shows a much slower transition from coal to CCGT, from a much earlier starting point of 25 GBP/tCO_2 to the same end point as in the Pure Dispatch Model (Figure 1) of 66 GBP/tCO_2*

In Figure 2 the composition of the simulated UK grid can be seen as the tax rate is increased. This composition may also be seen to significantly differ from that of the pure dispatch model, with a more gradual transition from coal to CCGT, rather than a single price determining the shift. It should also be noted, and will be analysed and discussed later, that this gradual transition is entirely before the cut off point from the pure dispatch model. This cut off point is at 65-66 GBP in both cases, beyond which point the contribution of coal power is negligible. Just prior to coal being entirely replaced in the pure dispatch case, the CCGT output is negligible also, however this is not true in the OPF model, where CCGT has already replaced just over half of coal production at tax-less levels.

3.2 Transition Window

In Figure 2 it can be seen that rather than CCGT fully replacing coal at the 65–66 GBP mark that it instead begins to be phased in to replace it from a much lower tax rate. Initially the rate of CCGT use is quite marginal below the 25 GBP mark, but at this point, at which the tax rate is still less than half of the transition point from the pure dispatch model, the use of CCGT begins to increase more substantially, already utilising 770 MW. So, while in the grid model there is already a 6.34% replacement of coal power with CCGT at 30 GBP/tCO_2, there is no replacement whatsoever in the pure dispatch (nationally aggregated) case, although the composition of grid power from CCGT is still quite low at that point. The difference between these two models becomes more pronounced, however, as the 65–66 GBP tax rate is approached.

Table 1: *Generator replacement by carbon tax rate (GBP/tCO_2).*

Tax Rate (GBP per tCO_2)	Regional Replacement (%)	Pure Dispatch Replacement (%)
25	≈ 0	0
30	6.38	0
35	10.78	0
40	15.24	0
45	19.59	0
50	25.06	0
55	33.14	0
60	41.53	0
65	54.78	0
70	100	100

For the tax rate range in which replacement of coal with CCGT power occurs, Table 1 displays the replacement of coal power with CCGT power at 5 GBP intervals. In all but the last column it can also be seen that using a purely economic dispatch model that no CCGT replacement of coal power in the portfolio composition occurs. Thus, all except the last column also represent the difference in the ten bus model's replacement rate to that of the one bus model. This rate of replacement can also be seen in Figure 3, where it is shown more clearly.

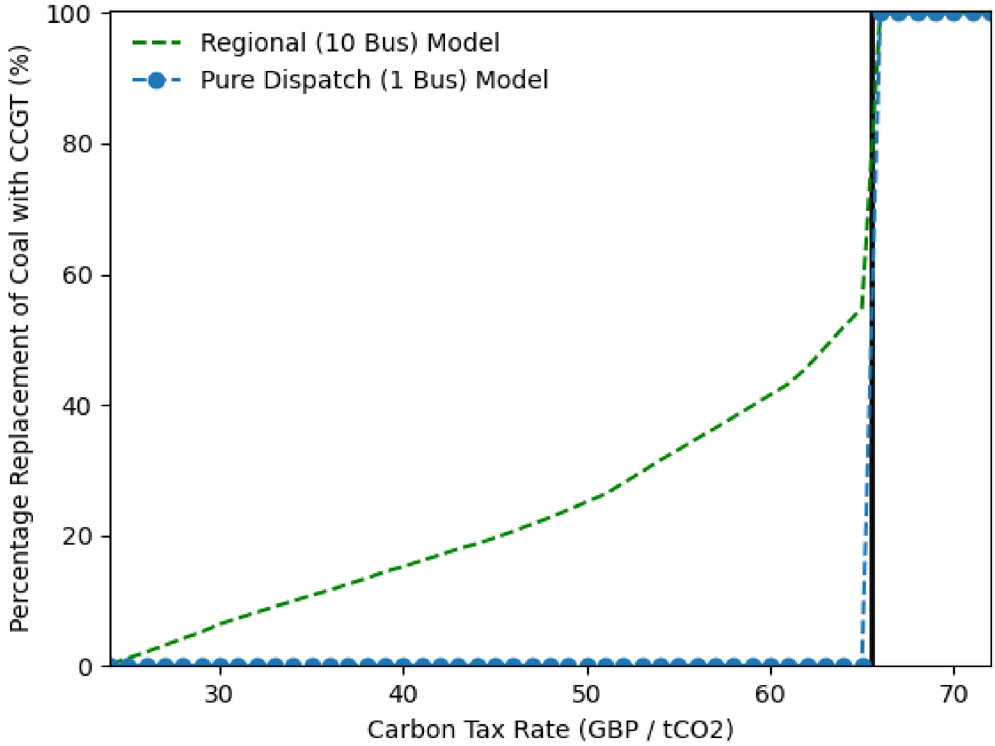


Figure 3: *Replacement of Coal Generated Power with CCGT Generated Power (Displays the replacement of coal with CCGT at increasing carbon tax rates). The marked line denotes the point where CCGT becomes cheaper than coal on the generator side.*

In Figure 3, therefore, it can be visualised that up until the transition point between 65 and 66 GBP, at which point the pure dispatch model entirely replaces coal with CCGT, that the difference in the replacement rate isn't merely significant, but grows to 54.78%, thus meaning that the majority of the replacement of coal with CCGT has already occurred by the transition point where CCGT becomes cheaper than coal in the grid model.

Furthermore, it should also be noted that although the difference between the replacement rates grows until the 65-66 GBP cut off point, that this effect is not merely a spreading out of the replacement of coal with CCGT. Thus, while some CCGT is phased in earlier in the grid model, there isn't also CCGT which is phased in later in this model. With all these results known they must therefore be discussed in greater depth to determine the causes of these phenomenon.

As the scope of this transition from coal to CCGT power occurs at above 24 GBP/tCO_2, that range will be the primary focus of the transition analysis. Other utilised power sources such as nuclear and renewable energy are used as much as is available due to their lower costs, so while they represent a component of the generation mix, as their contribution is constant, they will not be a source of focus in the analysis of this study. Finally, as the output levels of OCGT, biomass, and other generator types represent a negligible component of the power output due to their higher cost, they will also not be the focus of investigation.

3.3 Loss and Emission Reduction

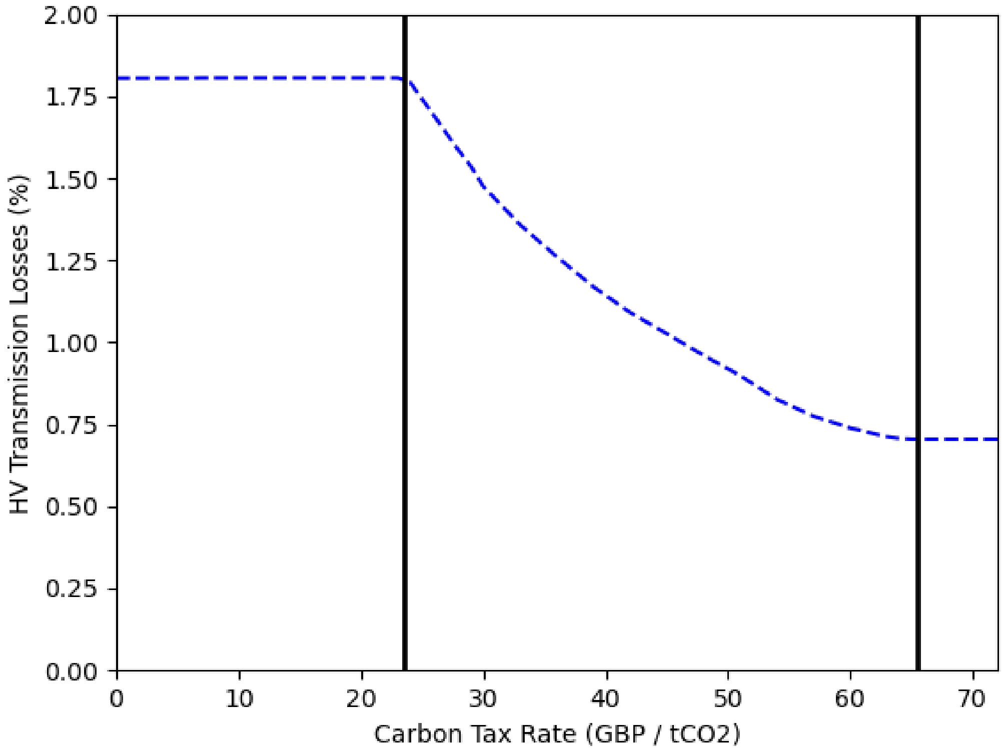


Figure 4: *Transmission Losses (Distribution and non-long-distance HV losses not considered). The transition window falls between the noted lines.*

In Figure 4 losses can be seen to more than halve throughout the transition of CCGT replacing coal. Initially (without a carbon tax), as coal is cheaper than gas, it is more economical to fulfil demand with coal power, even if it incurs greater transition losses; so as coal becomes more expensive, closer gas alternatives are utilised. Furthermore, as CCGT capacity is typically closer to demand, at very high tax rates losses are still lower than in the initial case.

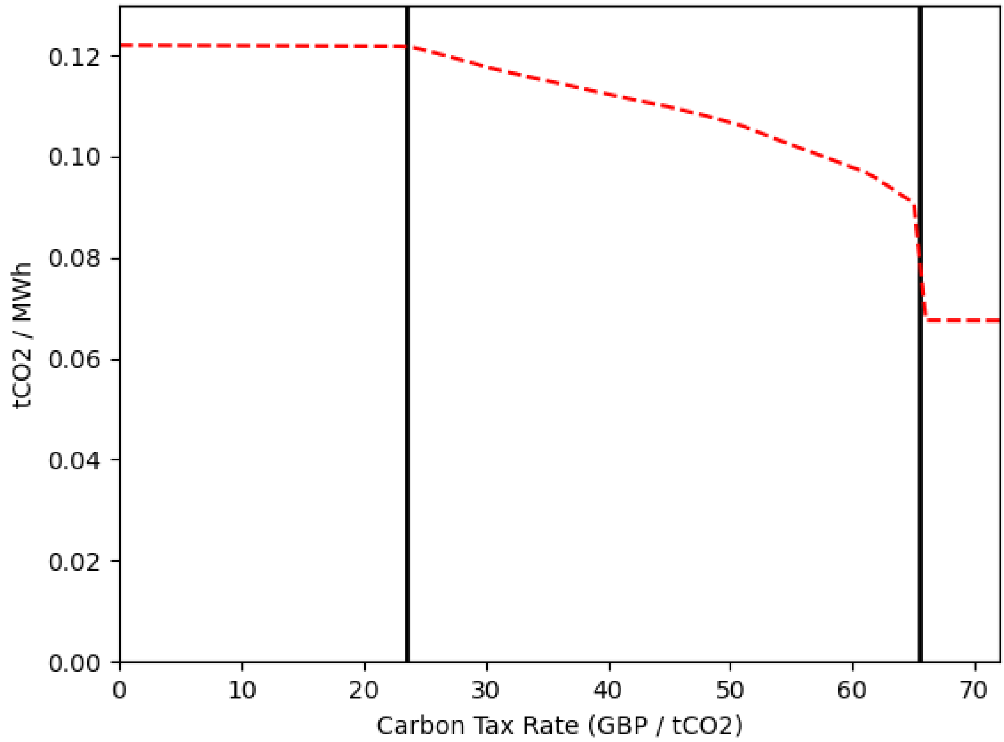


Figure 5: *Emissions per unit of power produced (from all sources including renewable and nuclear power, not just coal and gas) at different carbon tax rates. The transition window falls between the noted lines.*

Figure 5 shows emissions reducing as the carbon tax rate increases, as would be expected. Figure 5 displays the averaged emissions per unit of power produced from all major sources (renewables, nuclear, coal and CCGT). Once the tax rate reaches 24 GBP/tCO_2, where gas begins to be phased in to replace coal power, the emissions produced starts to decrease gradually until the end of the transition window at 65 GBP/tCO_2. Thereafter CCGT power becomes cheaper to produce than coal based on the carbon tax rate alone and thus all remaining regionally local coal power is replaced by local combined cycle gas, resulting in a steep decline in emissions. It should be noted, however, that as CCGT power still produces emissions, emissions caused by generation still occur, although the output at 66 GBP/tCO_2 represents the minimum level of emissions possible to meet average demand with the UK's present infrastructure.

4 Sensitivity Analysis

To analyse the integrity of this model a two part sensitivity analysis is performed. The first component evaluates the significance of OHL parameters while the second analyses the regional composition breakdown. It is determined in both that the underlying trends observed from the processed data remain the same even when these model inputs are significantly altered; thus the model is sufficiently insensitive.

4.1 Resistance Sensitivity Analysis

Resistance plays a clear role in power loss in this model. Prior to CCGT being cheaper from coal at the same location the reason why it is still phased in is due not only to the carbon tax, but also the complimentary transmission losses caused by OHL resistance. In Figure 6 the resistance is altered from the base case by 10 and 50 percent. Despite these changes the same fundamental trend exists (though starting much earlier when the resistance is higher and later when the resistance is lower).

These trends are at their greatest point of difference at lower carbon tax rates, where the effect of the complimentary transmission losses have a relatively greater influence. Then, as the carbon tax rate increases the effect of the carbon tax becomes more significant relative to transmission losses until it becomes a sufficient factor for replacement without the impact of transmission losses at all. Hence why the trends converge as the tax increases. As the same underlying trends this paper discusses remain even when significant changes are made to line resistances it is determined that the model is sufficiently robust with respect to line specifications.

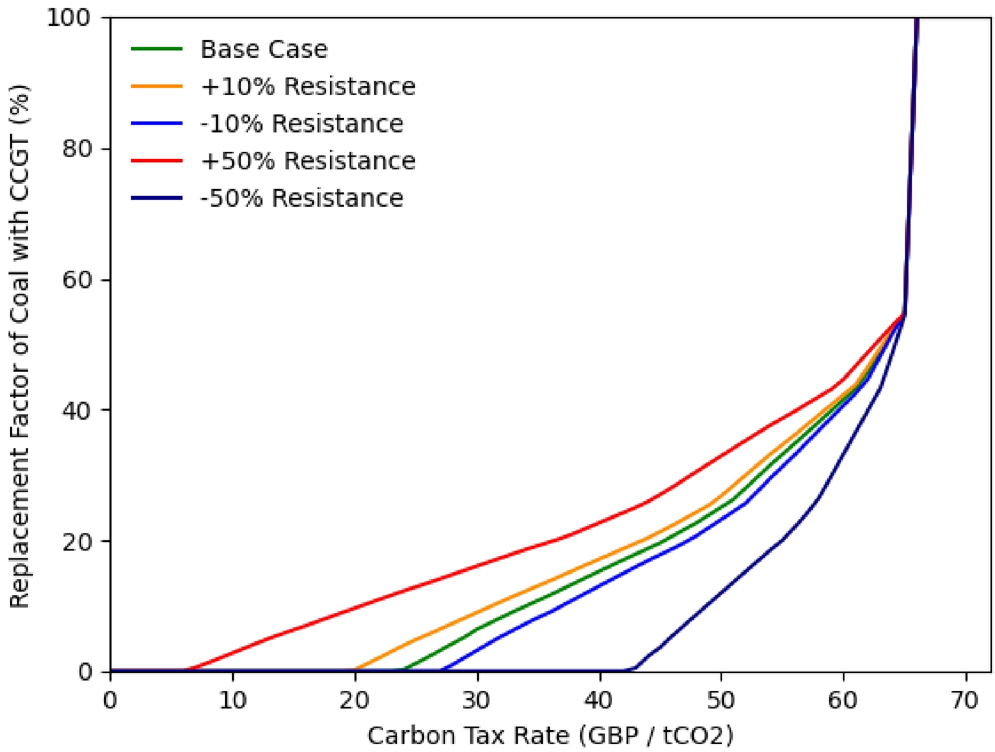


Figure 6: *Resistance Level Sensitivity.*

4.2 Regional Composition Sensitivity Analysis

The second component of this sensitivity analysis is regional composition. As part of the creation of this model generators were clustered into their geographic regions as defined in the methodology. The purpose of this section is to determine if the trend of coal replacement with CCGT persists even when differing regional structures are used and therefore verify that the trends observed are due to the fundamental characteristics of the grid, and not simply emergent from the choices in model construction.

To test the effect of how the regions were divided three alternate configurations are plotted. These differ from the base case by combining two adjacent regions and analysing the effect this has on the overall grid's transition. The first of these is the combination of the 'North West' with 'Yorkshire and the Humber (/ North East)'. As will later be discussed when specific regional characteristics are analysed, the north of England is a net coal energy exporter to southern regions. Thus, even though they are defined as separate regions they perform extremely similarly making the changes to the transition caused by this merge extremely minimal. In fact, if future models required additional simplification from what was used in this study, the combination of the English North would provide the least distortion to results. Thus, as these regions already behaved near identically, their merge caused an insignificant impact on the model's results; seen in Figure 7.

The second merge will be between the South East and London. Both of these regions are initially reliant on coal power from external regions, but transition to using local (and in the case of London, imported) CCGT power to fulfil local demand. As the South East has sufficient CCGT capacity for both regions, the combination of these regions effectively removes the transmission losses between London and the South East, making CCGT power replacement of coal (which is imported at a greater transition loss) occur at an earlier stage. Thus, while CCGT transmission losses are typically lower, even in the base case due to generators being closer to demand, the combination of these regions increases this effect, making the transition occur from an even earlier stage. While the effect of this regional merger is more significant than that of merging the Northern English regions, it can still be seen to clearly follow a similar transition path to the base case (Figure 7).

Finally, the East and East Midlands are combined. The East Midlands is a substantial coal power exporter to the East (and South), however the East has local CCGT capacity. Thus, in the base case, at a point where the combined effects of the carbon tax and transmission losses from the East Midlands to the East outweigh the cheaper base cost of coal, local CCGT capacity in the East replaces coal imports from the East Midlands. With these regions combined this transmission loss is not considered and thus the transition occurs at a later point, hence the delayed transition in this case. Although this significantly effects the East's transition, once again, the same overall trend is observed as in the base case, even if it is delayed. Thus, this sensitivity analysis concludes that the regional breakdown of the UK used was sufficiently robust and that these trends are not simply a byproduct of arbitrary model construction decisions.

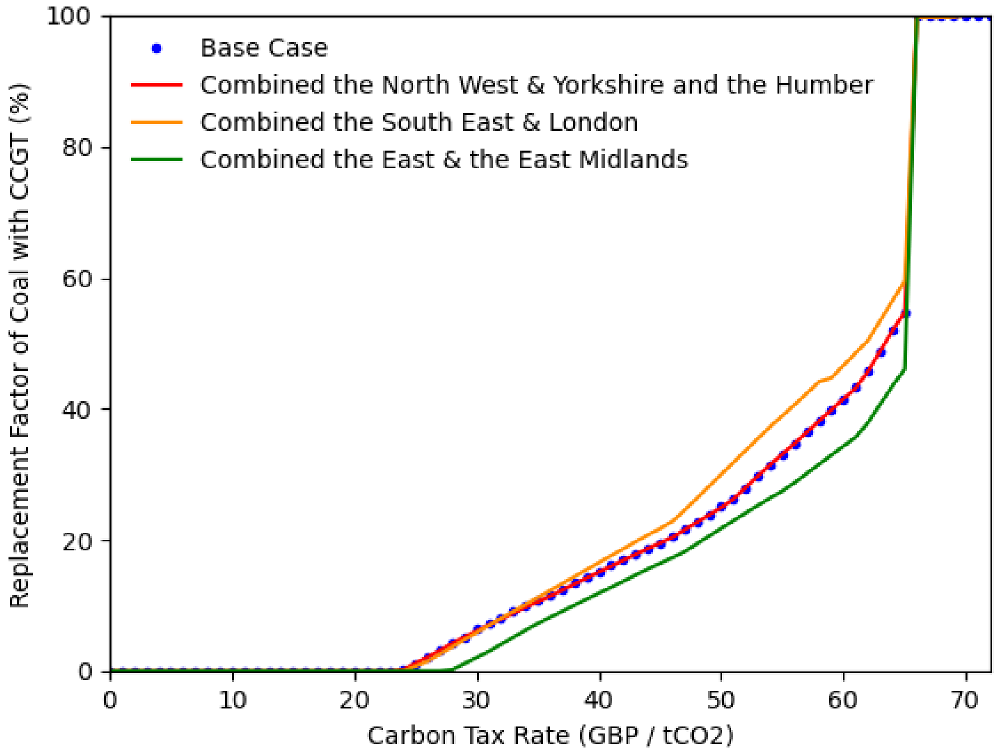


Figure 7: *Regional Composition Sensitivity.*

5 Carbon Tax Analysis and Discussion

The introduction of a grid model (resulting in an OPF model) in addition to a pure economic dispatch model produces significantly different results. These specific difference in question refers to the replacement of coal power generation with combined cycle gas turbine power generation, which is deemed to be definitively significant and take place predominantly between the tax rates of 30-66 GBP/tCO_2.

While this window of tax rates encapsulates the most significant component of the OPF transition, the transition period in the pure dispatch model occurs entirely between the tax rates of 65 and 66 GBP, as this is when the carbon tax impact on the cost function makes CCGT power production cheaper than coal from the same location. While the cost functions for each generator type were the same across both models, the distinction between the models was the number of busses and their interconnecting branches, meaning that the cause for this difference between the two models was the inclusion of a grid model.

In summary, the pure dispatch model transitioned entirely from coal to CCGT at a tax rate of 65.2174 GBP/tCO_2, while the OPF model transitioned from coal to CCGT over a much longer period, from <30-65.2174 GBP/tCO_2. It should be noted that not only does the transition take place over a longer window of time, but that the lower bound of this transition is at a significantly lower tax rate than in the pure dispatch model, while the upper bound remains the same. The purpose of this discussion will be to consider not only the broader window of the transition, but also why this window results in a significantly reduced lower bound, while the upper bound increases by no such margin and, in this model, remains at the same level as in the pure dispatch case.

5.1 Broadened Transition Window

Of these findings the more obvious of the two to explain is the broadening of the transition window; which is due to the inclusion of the grid model. While the same base cost function (factoring in size) is the same for each generation type, the inclusion of grid conditions means that due to there being potential losses of different magnitudes between generation and demand, that the while the costs of generation remain the same, that the costs of meeting demand now differ.

This effectively changes the costs of each generator with respect to each load in the system, as there will be a specific loss associated with the transmission between these points. This effectively alters all the costs of meeting demand by differing amounts, meaning that the point at which it becomes more economical for gas to replace coal is no longer a single price, but a variety of prices based on the specifics of each generator and load.

Furthermore, but it should also be noted that loads will not only be drawing power from generators at different locations, but furthermore, that it is possible that the replacement gas generator will be from an entirely different location, or potentially multiple locations simultaneously based on however the OPF model is able to minimise the cost of meeting demand. Even using a regional bus allocation as is the case in this model, there remains a mismatch between demand, coal capacity and CCGT capacity, meaning that it is a necessity that the model not only draws from other regions, but also that the regions it is most economical to draw power from will change as the carbon tax is increased.

This can be seen in the following table as well as being visualised in Appendix B:

Table 2: *Averaged Hourly: Load (MW), Coal Capacity (MW) CCGT Capacity (MW) and Total Capacity (MW)(*,**); by region (***).*

Region	Load	Coal	CCGT	Total
South East	4305.753	35	4799	10572.63
South West	2690.714	0	2,969	7,979.9
London	4349.412	0	408	645.4
East	2997.804	0	3,714	9,383
East Midlands	2354.482	6,020	4,780	12,711.7
West Midlands	2730.947	0	0	503
North West	3493.016	1,961	1,825	9,552.5
Yorkshire & the Humber & The North East	3876.755	3,940	3,476	15,971.4
Wales	1696.432	1,816	5,680	12,767.4
Scotland	2756.565	0	1,180	12,248.7

** Total capacity includes the capacity of all power sources (including tax unaffected nuclear and renewable), not just CO_2 emitting coal and CCGT.*

*** Non-dispatchable renewable capacity within total capacity is not indicative of the renewable output as the model presumes a fixed average output.*

**** Regions are consistent with those defined in section 2.*

Given the variety of generator compositions, loads and allocation of coal and CCGT generators the grid must be utilised to resolve the dispatch problem. As a result the cost of meeting demand, despite the use of the same cost function coefficients for each generator type, the point at which it is economical to replace coal with CCGT varies. These differences should be discussed and are as follows.

Some specific points of note are as follows. Firstly, the West Midlands bus, which contains a large load, but no substantial generation of CCGT or coal. Secondly, a number of busses, particularly the London bus (though also the South West, East and Scotland busses), contain a high load, yet no coal power generation. Also in the case of London, there is only a minor amount of local generation, of which most is CCGT. Furthermore, the closest (and lowest loss power transfer factoring in HV lines) bus to London, the South East (which if it were to gave excess generation capacity would therefore most cheaply export to London), contains only a very minor coal capacity, with much higher CCGT generation capacity.

The implications of phenomenon such as these will be discussed in greater depth in Disproportionately Greater Broadening of the Transition Window's Lower Bound section. From the results thus far, however, it has been determined why the transition window from coal to CCGT is broadened.

5.2 Disproportionately Greater Broadening of the Transition Window's Lower Bound

Due to the variations in the grid and generation configuration of the UK's grid it has been determined that the transition window from coal to CCGT has been broadened, however, this has not occurred in equal measure at the lower and upper bounds of that window. Notably, CCGT power begins being phased in at a much lower tax rate. It must therefore be determined why this disproportionate effect of broadening exists. As this phenomenon is due to the inclusion of the simplified transition grid into the model it will therefore assist in explaining this phenomenon to graph the coal and CCGT power generation by bus to geographically breakdown this transition.

5.3 Overall Transition Breakdown

Considering the transition (24-66 GBP/tCO_2) from coal to CCGT on a regional level shows the different behaviours of these areas.

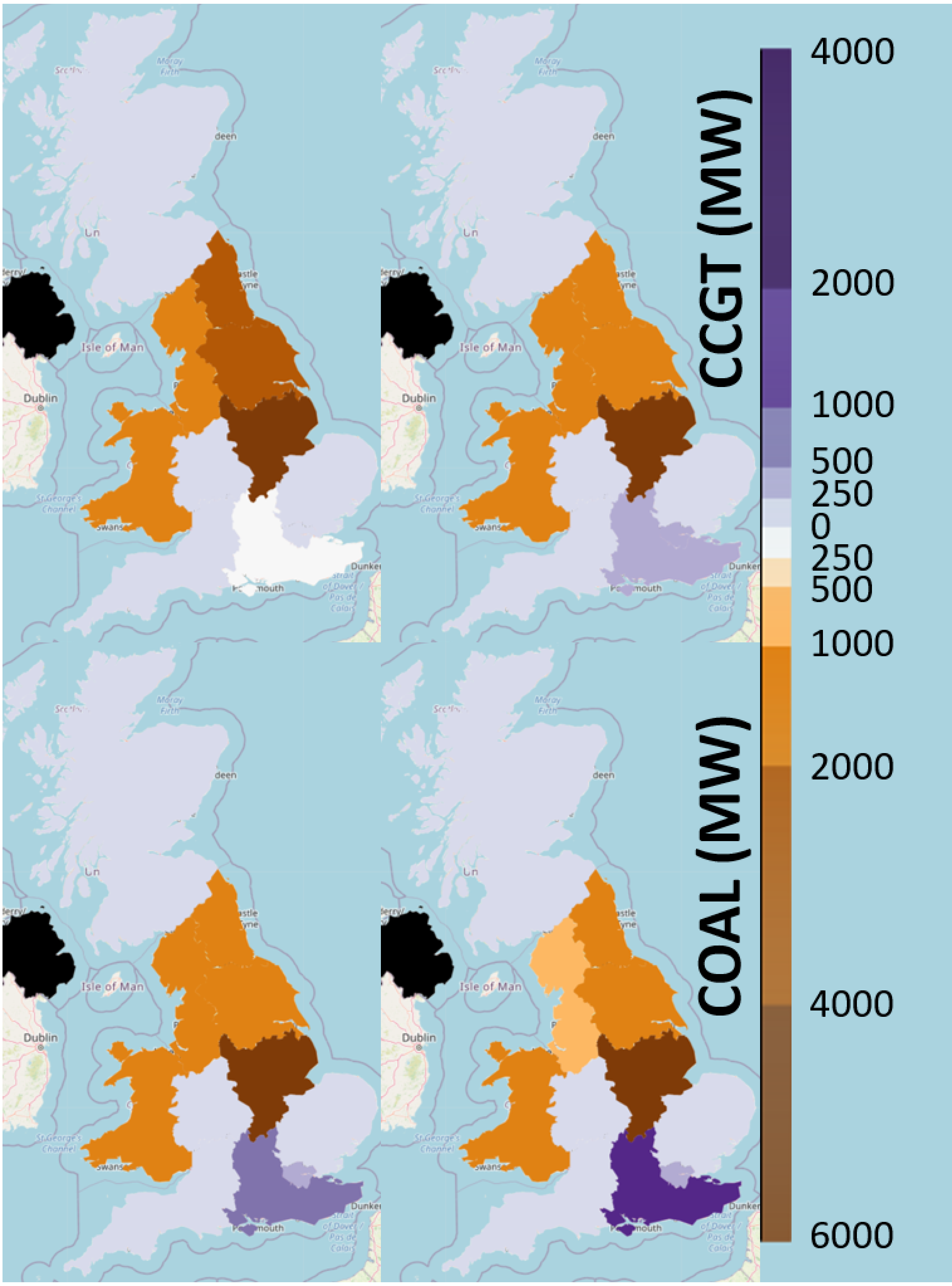


Figure 8: *Regional Coal and CCGT Output (Top Left: 24 GBP/tCO_2, Top Right: 30 GBP/tCO_2, Bottom Left: 36 GBP/tCO_2, Bottom Right: 42 GBP/tCO_2. Note for these shapshots that within a region coal and CCGT are not run simultaneously, allowing for both outputs to be shown at once.*

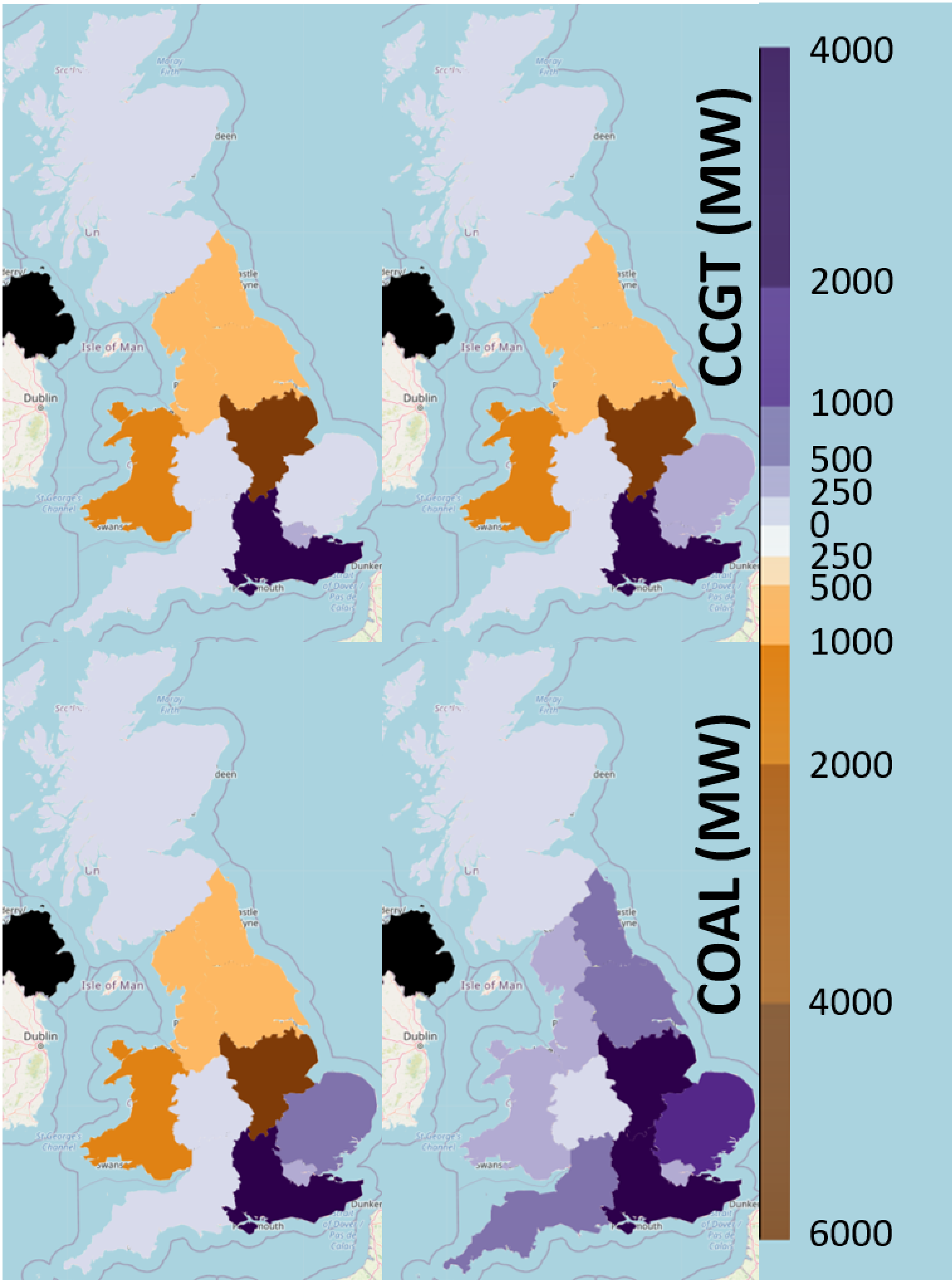


Figure 9: *Regional Coal and CCGT Output (Top Left: 48 GBP/tCO_2, Top Right: 54 GBP/tCO_2, Bottom Left: 60 GBP/tCO_2, Bottom Right: 66 GBP/tCO_2. Note for these shapshots that within a region coal and CCGT are not run simultaneously, allowing for both outputs to be shown at once.*

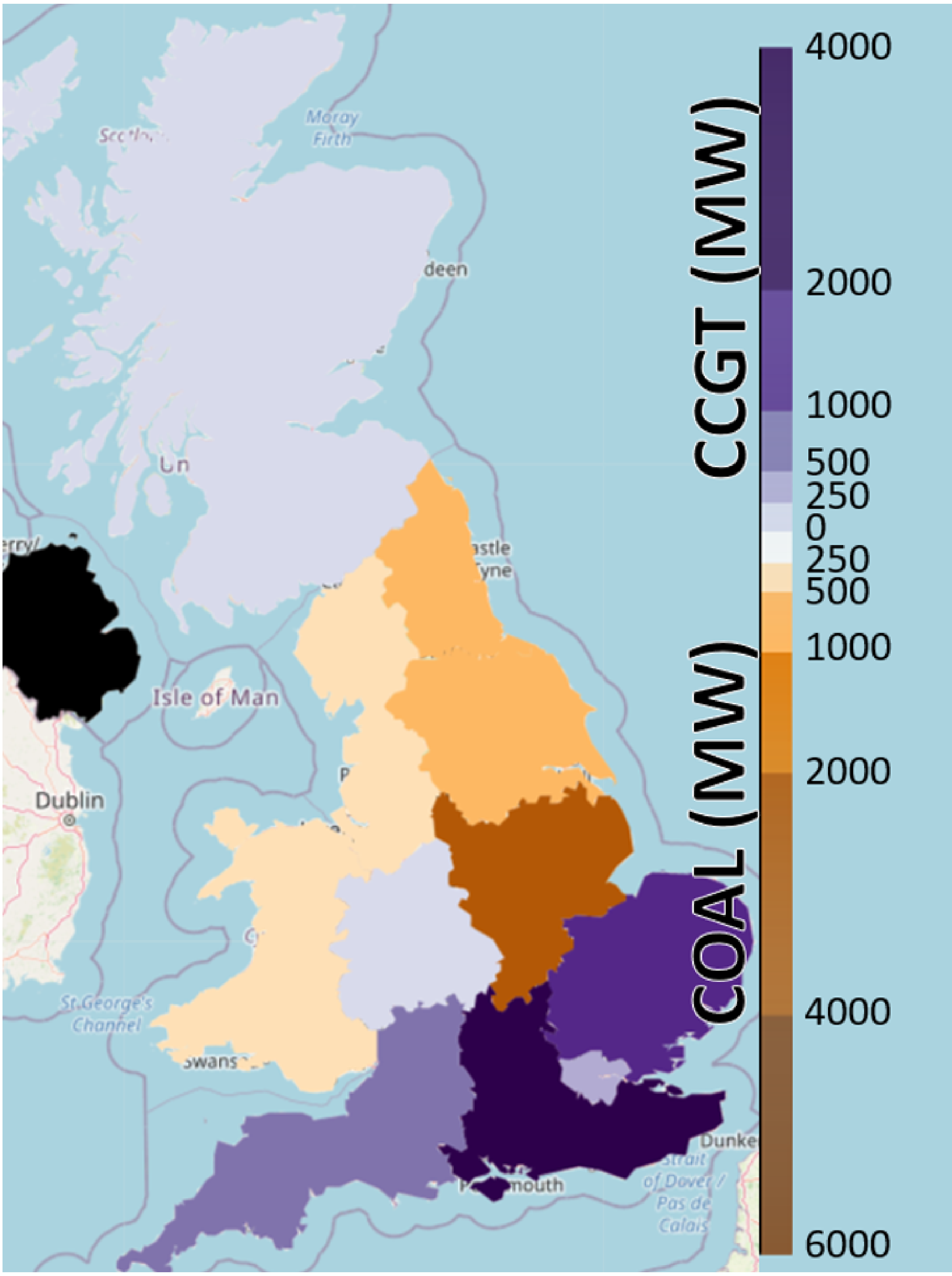


Figure 10: *Britain's Regional Power Generation Compositions at their maximum point of difference from a nationally aggregated model (Carbon Tax Rate of 65 GBP/tCO$_2$). Note that coal and CCGT are not produced simultaneously in a region; allowing a shared scale.*

In Figure 8 and Figure 9, eight snapshots of the transition from coal to CCGT based generation are shown. The third figure (Figure 10), occurs at a tax rate of 65 GBP/tCO_2, showing the generation mix just prior to CCGT becoming more affordable than coal due to the influence of the carbon tax alone; the effect of which can be seen in the fourth (66 GBP/tCO_2) sub-figure in Figure 9). It should be reiterated that the regions shown are similar to those generally used to analyses of the UK, but with the North East and Yorkshire and the Humber merged into a single region for simplicity, and Northern Ireland not included due to its greater external power grid integration. Furthermore, the same axis can show both coal and CCGT power generation as neither are produced simultaneously in a single region in the analysed carbon tax window as stated in the figure description. Finally, nuclear and renewable energy outputs, as they are constant, are not shown, though they continue to play a significant role.

Figure 11 is taken at a tax rate just prior to the tax rate making **local** CCGT cheaper than coal. Thus it represents the point just prior to the most rapid point of change (from 65 to 66 GBP/tCO_2). This figure is used to display all generation types, not just coal and CCGT. For reference, if regional transmission losses were not included, then Figure 11 would contain no CCGT, instead utilising coal power from generators in the UK.

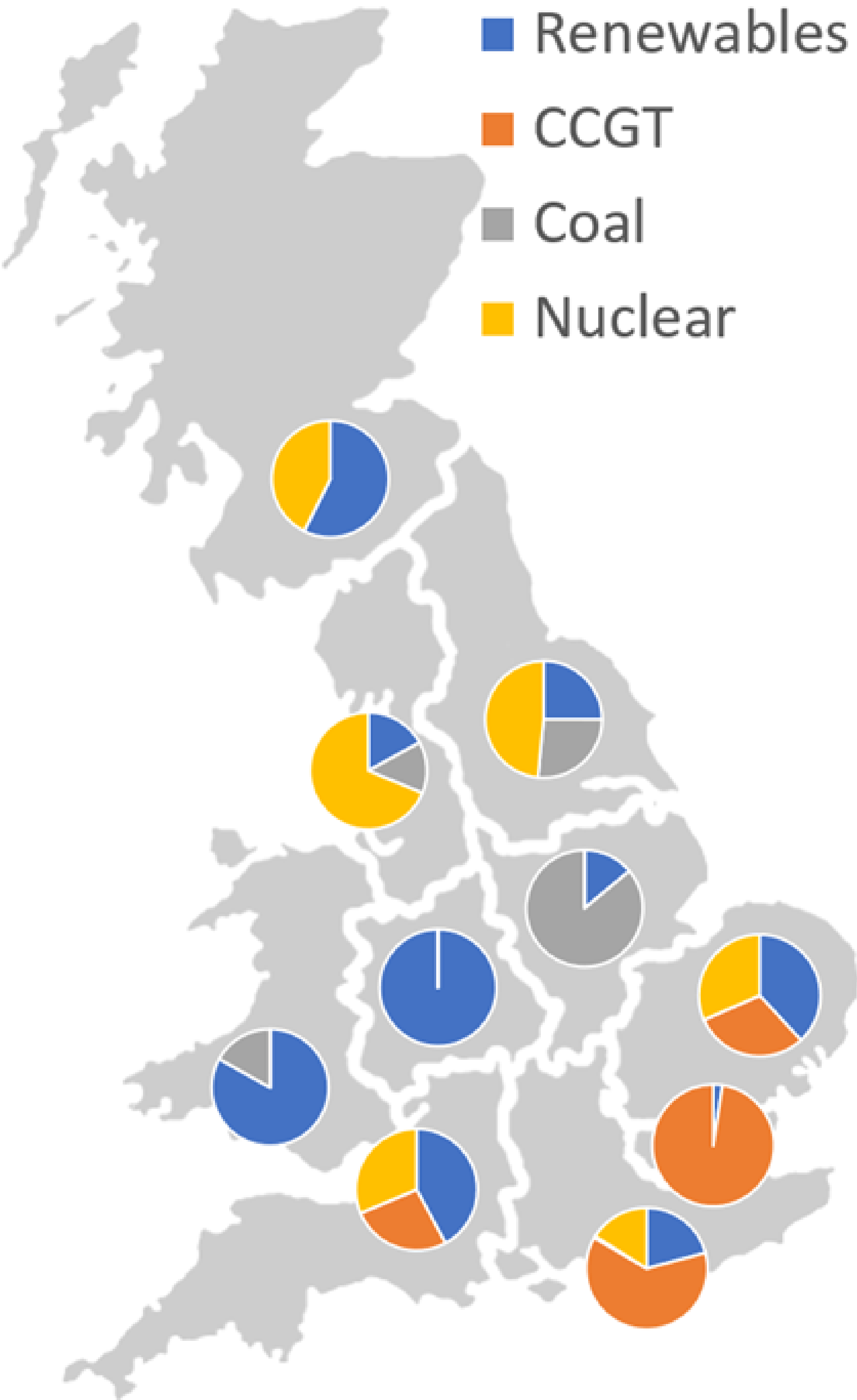


Figure 11: *Regional Power Generation Compositions (at their maximum point of difference from a model which doesn't consider regions; with a non-regional model using coal in place of CCGT entirely at the same carbon tax rate).*

The visualisations in Figures 8, 9, 10, and 11, (the latter two of which occur at the same tax rate) allow convenient observation of regions' transitions. Figure 11 in particular allows the significance of the coal vs CCGT transition to be seen in the perspective of the overall generation mix.

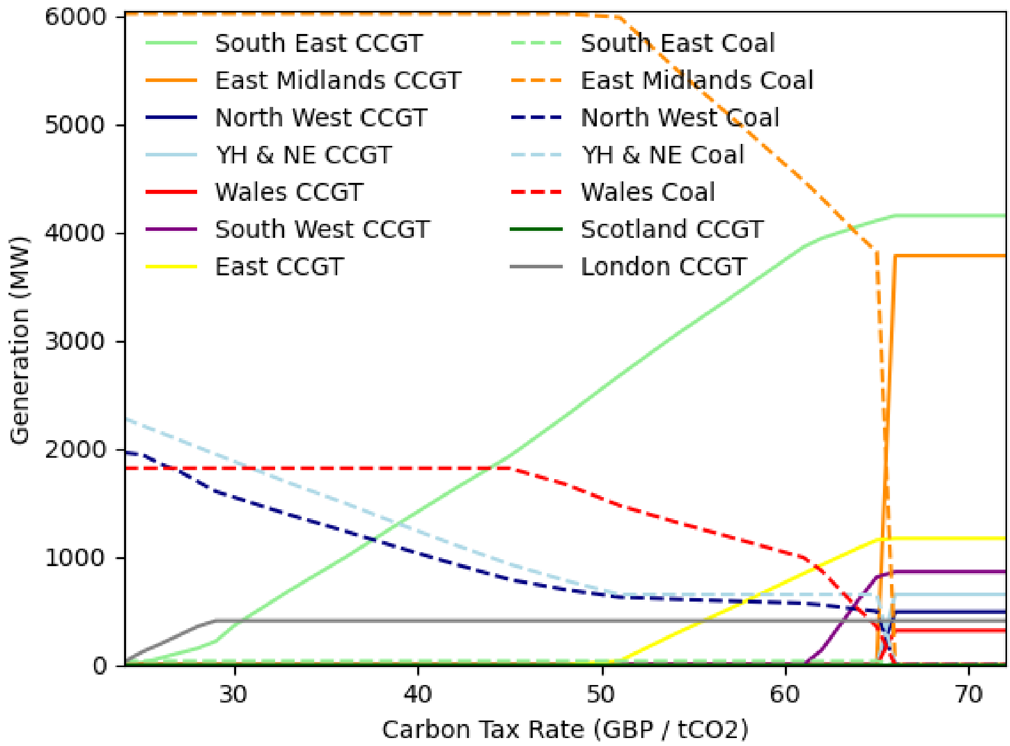


Figure 12: *Displays the coal and CCGT power output levels (MW) by region. Each region has a colour, with CCGT being represented by solid lines and coal by dashed lines. An overall transition from coal to CCGT can be shown, particularly with northern regions' coal power being replaced by CCGT generation in southern regions.*

In Figure 12 each colour represents one of the regions, with dotted lines representing coal generation in a given region and solid lines representing CCGT. The transition from at 65–66 GBP/tCO_2 shows the final point at which coal's contribution to the generation portfolio is significant, before being fully replaced by CCGT. In particular, regions with insufficient local coal generation capacity can often more affordably utilise local CCGT, or CCGT from a region with a lower transmission loss than the region they would otherwise draw coal power from.

5.4 Regional Transition Breakdown

CCGT being preferred over coal production due to lower transmission losses and thus being phased in earlier occurs most notably in the South East of the Country. This is not only in the South East region, but also in London, which serves as a good starting point for regional analysis. London has no local coal production, relying on coal power imported from throughout the country initially, but by the 30 GBP/tCO_2 mark it has already fully deployed its local CCGT capacity to assist in meeting local demand. As the carbon tax continues to rise London relies less on coal imports from more distant regions (where coal power production is concentrated) and begins to import power from the South East's CCGT generation.

At the start of Figure 12's displayed information London's CCGT output can thus be seen to be the highest of any region, but due to its local capacity being insufficient to meet London's demand, power imports from the South East dominate London's power consumption as the tax rate is increased. The South East also, follows a similar trend. Initially its CCGT is only partially deployed to meet demand both locally and from London given that its local coal capacity is far too small to meet either of these requirements. As the tax rate is increased the large CCGT capacity of the South East is thus increased, switching the region from a power importer to a power exporter. Throughout this, however, it should also be noted that the South East's minor coal capacity remains utilised until being fully replaced with CCGT at the economic dispatch replacement price (65-66 GBP/tCO_2).

This same phenomenon occurs in the East, which having coal capacity of its own is initially reliant on power imports, before the combination of the carbon tax and transmission costs (in the form of losses), combine to make local CCGT power more affordable. Thus, from 51 GBP/tCO_2 onwards its CCGT output can be seen to increase until CCGT power becomes cheaper than coal regardless of transmission losses (beyond which point CCGT has fully replaced coal).

The net result of these effects is that while the UK's coal power is dispatched predominantly from the UK's East Midlands and North. As the tax rate is increased, due to the transmission losses from these regions to the south first England's northern regions (from which the transmission distance and losses to transfer to the south are the greatest), the coal power output from these regions is the first to be substituted, falling as the South East's CCGT output increases. Wales's coal output is the next to undergo its decline, followed next by the East Midland's massive coal power output, which is the closest of the major coal regions to formerly coal power importing London, East and South East regions; thus being the last major coal capacity regions to begin its decline in coal power production.

As the coal imports from the North and Midlands/Wales are reduced, they are instead met by lower transmission loss (closer) and local CCGT power. London is the first to make this transition, having done so by the start of the main window of analysis, though its generation is insufficient to meet even local demand. The South East, East and finally, the South West are next to follow in increasing their CCGT output, having sufficient capacity to meet far more of the South and East's demand.

Despite transmission losses and the carbon tax both incentivising the successful replacement of approximately half of the coal capacity of Britain with CCGT, however, coal demand persists until the carbon tax alone can incentivise its replacement with CCGT. Thus, where power imports from the North and Midlands are still required, (as well as in small part locally in the South East), as transmission losses from CCGT and coal remain the same, the carbon tax's incentive alone must incentivise the replacement of coal in these instances. The most notable example of this is in the East Midlands, where, while imports do reduce, they still remain substantial, resulting in reduced, though still high coal power production until the point of full replacement at the economic dispatch replacement tax rate (65-66 GBP/tCO_2, where even without transmission losses, CCGT becomes cheaper).

Finally, it should be noted that as coal and CCGT do not fully comprise the entirety of power generation, the inclusion of nuclear and renewable energy sources means that the demand being discussed in this section is the remaining demand after the usage and potential transfer of renewable and nuclear energy. For this reason it should not be surprising that the coal power output from many high coal capacity regions drops below the level of local demand, as some of that local demand is being met by nuclear and renewable generation either locally or from other regions (such as in Wales and the North).

Scotland, furthermore, has a very high level of renewable power generation and thus requires little power from either coal or CCGT. Thus, in cases where coal and CCGT are more prominently required (such as in the high demand regions of the South, especially London), the transition is more pronounced and thus the results far clearer to interpret. Similarly, net power production (coal, CCGT, renewable and nuclear power) will also be higher than power demand due to the inclusion of losses, though this is far more less significant, and ultimately negligible given that loss minimisation is a component of OPF cost minimisation.

In summary, the replacement of coal power with CCGT is therefore done to meet the demand of busses who's demand is unfulfilled by local coal power, with these busses often having no local coal power or a lower level of local coal power capacity than their demand (after accounting for renewable and nuclear power flows). Thus, although CCGT is still more expensive than coal to produce a MW on the generator side at tax rates <65.2174, as CCGT generators are closer to these loads than coal power, factoring in transmission costs it is cheaper to meet this demand with CCGT power.

This same phenomenon was already discussed in the previous section, though given the lower concentration of coal power in specific regions, CCGT power, particularly in the high demand south and east, incurs lower transmission losses than coal in meeting demand. This is reflected in Table 2, where there are regions with CCGT, but not coal, though none with coal and no CCGT (or with extremely minor coal capacities such as in the South East).

It is therefore due to CCGT generators being more spread out throughout the country as well as them being closer to the points of high demand that the broadening of the transition window from coal to CCGT results in a disproportionate effect on the lower bound of the window. If we were to consider the tax rate of 30-66 GBP/tCO_2 to be the transition window for the grid model (which may be somewhat generous as London's CCGT power is transitioned in from a very low tax rate, though this process being more gradual makes its significant starting point less clear than the cases shown in Figure 12), then this shows the lower bound is more than halved than in the pure dispatch model where the window is 65-66 GBP/tCO_2. Thus, the cause of the disproportionate broadening of the transition window has been determined and is to the benefit of CCGT.

6 Further Discussion and Recommendations

This model finds that CCGT replaces coal from an earlier carbon tax rate when inter-regional transmission losses are considered; as CCGT sources are closer to the major loads. The model used to reach this conclusion simplified the grid into a 10 bus model with identical cost functions for each generator type. Thus, to add further clarity to this finding as well as more specific detail to specific cases underpinning this transition, specific cases throughout the transition will be discussed in this section. This discussion will occur on a more specifically regional level, analysing some key cases present in the overall data.

6.1 Transmission Model Constraints

While the model used as determined to be both sufficiently complex and robust, even if these simplifications are reasonable, for the purposes of completeness, their implications should still be disused. While all major generators had individual instances, the main simplification made by this model is the regional grouping of loads and generators and the associated branch network.

Due to the regional grouping of this grid data, transmission losses within regions are not considered. Also, while inter-regional power transfers are considered, they are simplified to a single bus in each region, while in practice a broader range of losses from generators to loads would exist based on their positions within their respective regions. While load and generator locations were regionally clustered, however, the comparison with other simplified configurations in the Sensitivity Analysis demonstrates this generalisation to not undermine the robustness of the model. Thus, while generalised, a regional level of resolution was sufficient in capturing the effect of more dispersed gas generator placements throughout the nation, particularly in the East and South (especially in London and the South East), where the regional placement of generators has its greatest effect due to the closer proximity of CCGT to southern loads.

Despite the resolution of this model therefore being consistent with others at similar levels of resolution, as well as sufficient to capture the most significant effects of grid conditions, the transition window may still be affected in practice by intra-regional conditions. Specifically, while in this case there is no load closer to a coal generator than a CCGT generator, in practice, there may be minor exceptions.

Although the greater dispersion of CCGT would still likely mean these cases would be outnumbered by cases of closer CCGT production, there may be a minor increase in the upper bound of the transition window. Similarly, it is also possible that a lowering of the transition window occur due to this effect (especially in combination with inter-regional transmission losses), though this distortion would remain minor compared to the inter-regional effects noted in this investigation.

Therefore, in the same way that transmission losses incurred by northern coal plants exporting to southern loads were a more significant influence on the transition from coal to CCGT than the losses incurred by northern coal plants exporting to northern loads, the consideration of intra-regional effects, the effect of intra-regional transmission losses would be far less significant than that of inter-regional grid conditions.

In summary, while the model resolution is appropriate in performing this investigation, as grid effects are at their most prominent over greater distances, given that some minor effect on the transition window is possible due to intra-regional conditions, in practice it would be more accurate to say that this investigation concludes that the broadening of the transition window occurs disproportionately in the reduction of the lower bound, rather than exclusively on the lower bound, as there may be some minor cases where coal power would incur lower transmission levels to meet demand somewhere in the grid compared to CCGT produced electricity.

As such, in the trade-off between the model's complexity and integrity (as greater detail could increase the risk of failed convergence), the level of detail decided upon in this investigation both results in a robust model as well as capturing the prominent, inter-regional grid conditions' effect on optimal power dispatch. Finally, it should be noted that other simplifications, such as source type cost function assumptions and OHL ratings may also have effects of their own in practice, but that the scale present on an inter-regional scale dominates more minor variations. Thus, while variations of specific details would be suitable for a smaller scale, higher resolution model, the regional resolution utilised in this investigation is suitable for a national level analysis.

6.2 Policy Modelling Recommendations

With a discussion of simplifications in mind, other potential policy influences such as feed in tariffs, fees for specific generation types, carbon taxing of non-generation emissions, *etc.*..., were not considered as this analysis was used to study the influence of a carbon tax specifically.

Given that this study determines the effect of grid conditions, particularly on a regional level, to have a significant influence on impact of national energy policies (such as in this case where the effect on the present power portfolio was performed), future investigations into alternate emissions reduction legislation proposals may also benefit from the inclusion of a grid model due to the substantial effect it was found to have on the modelled results in this investigation.

Expansions upon this investigation may also be performed with regards to the OPF's construction. A more detailed breakdown incorporating additional buses and branches may also be performed, especially if such an investigation were to be performed on a smaller scale, such as investigating an individual region. Furthermore, while larger fossil-fuel based power plants are often further from population centres in order to further distance their greater pollution output and therefore would have greater intra-regional transmission losses, they often also produce power more affordably, being capable of producing it more cheaply in greater quantities. As a result, the use of more specific cost functions may also be prudent in such cases, either on an individual power plant level, or for power plants of different sizes or other parameters likely to have a significant impact on cost.

Finally, the results of this investigation also bare relevance to other types of power system analyses. It was determined that on a regional resolution, model behaviour of the South and East, Midlands and North exhibited similar trends due to their similar generation capacity compositions. Just as it was determined that a regional resolution proved to be useful on a national level of analysis and that a number of generalisations may require expansion in the event of a smaller scale analysis, an analysis on a larger scale seeking to create a more simplified model of the UK may benefit from the consideration of the similarities between the trends exhibited in these regions as a justification for their aggregation. A European model, for example, seeking to determine in what way to generalise the UK's grid, would therefore make use of these findings to ensure that the major inter-regional effects persist as much as is reasonably possible given the level of resolution utilised, by aggregating regions on this basis.

7 Conclusion

This paper analysed the effect of a carbon tax on Britain energy generation composition via the use of a 10 bus OPF model the UK's power plants, transmission system, and loads. This model was used to analyse the impact of a carbon tax on the generation technology composition (i.e. generation from coal, CCGT, renewables, nuclear, *etc...*) of existing power infrastructure to meet demand at minimised cost (factoring in transmission constraints). A regional OPF model is not only utilised to perform this analysis but is specifically analysed in comparison to a nationally aggregated model to further scrutinise the importance its selected resolution and the significance of regional power infrastructure differences.

To meet this objective a regional and single bus model were both constructed and their power output compositions by source type analysed at different tax rates on generation emissions. Using this process, it was determined that nuclear and renewable energy were both fully utilised, while the remaining power demand was met initially by coal power, but increasingly sourced from combined cycle gas turbine generation; other sources contributing only negligibly. This transition window was found to be broadened significantly (the lower bound more than halving) in the regional model, with this effect being disproportionately on the lower bound. By investigating the specific regions' individual transitions this effect was specifically attributed to regional generator compositions and the less concentrated placement of CCGT generators throughout the UK. Thus, introducing regional resolution does not merely add another cause for the variation of results and thus the broadening of the transition window, but that due to the specific trends in the placement of coal vs CCGT generators, that this broadening occurs disproportionately on the lower bound of the transition window. The regional division was further scrutinised by verifying the convergence of its results with further regional aggregations.

This analysis draws several conclusions about the significance of implementing a regional framework as opposed to an aggregated national model, specific regional trends present, and the effect of the OPF model on the composition of the UK's generation at an increasing carbon tax rate.

The notable conclusions of this paper are therefore that:

- The inclusion of a grid model has a significant impact on the modelled result of a national policy such as a carbon tax on power generation methods, thus regional power transmission is deemed to have a significant impact on the generation composition;
- Grid losses in the UK are found to provide combined motivation with carbon taxes to replace coal power plants with CCGT generators;
- Consideration of regional characteristics does not only broaden the tax rate window during which coal power is replaced with CCGT generated electricity, but it disproportionately broadens the window via the lowering of the lower bound of the transition;
- The cause for the broadening of the transition window is dominated by the greater dispersion of CCGT capacity compared to coal and its closer proximity to high demand areas, which is most present in the unique output capacities of different regions (such as the high coal capacity north England and East Midlands, contrasted by the high demand, low coal and more CCGT equipped South and East), and,
- That not only does the consideration of regional transmission and generator compositions have a significant effect on the modelling of national energy policies such as a carbon tax, but that this resolution is particularly justified in capturing the effect of multi-regional differences, further determining regional similarities which may serve as a basis for a more simplified model to be created while still reasonably capturing this effect if a larger scale / lower resolution model were to be constructed in the future (for example, in the case of an aggregated English North).

These conclusions serve as valuable bases for investigations into the UK's (and potentially other nations') power grid(s), the modelling of energy policies such as carbon taxes, and, future power plant placements. This study concludes that regional distinctions, particularly between the North / Midlands and the South / East, are significant and must be considered in policy and studies pertaining to affordably (and environmentally) meeting power demand. As both the carbon tax rate and generator placement with both determined to be significant factors in the replacement of coal power with CCGT generated energy, policy makers and planners seeking to make new constructions of more environmental generators should be sure to consider the impact of regional placement.

Research data

Research data supporting this publication is available in the University of Cambridge data repository (doi:10.17863/CAM.59414).

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A Appendix: Model Formulation

To input the required data for these calculations to be performed, into the computer model (PyPower or MatPower), data may be defined with reference to buses, branches and generators [13]. Of the various permitted input parameters, the following were used for each of these components for the purposes of this study. For the UK's high voltage transmission system a base voltage of 400 kV and a base power of 100 MVA are used.

A.1 Scope and Resolution

From the reviewed literature it was determined that a model making note of every specific piece of infrastructure, while utilised small scale analyses, is not suitable for a national or multinational model. As such, a level of generalisation is required to simplify the UK's grid. Furthermore, as this model considers the transmission network, Northern Ireland will not be considered in this analysis, with the more internally interconnected island of Britain being the focus of this analysis.

Load data is released on both an administrative district and regional basis, the former of these having a far higher resolution that would be required. The latter of these, which divides the Britain up into 11 regions, is much more suitable for an analysis (reduced to 10 in the final model as discussed in A.2). Transmission data is available in both the form of individual assets and a general visual map, which would have to be simplified to suit a generalised national representation. Individual generator data is also available for generators above 30 MW in capacity, with only some of those below that output being available. Given that an individual bus may have multiple generators attached and that this study focuses primarily on generation consumption, the simplification of the generator composition of each region is far less important than generalisations required for the transmission network. Thus, while there are 10 buses used, attached to them are a total of 275 generators. Finally, cost functions for these generators would be required, with these differing significantly based on the generation type and facing further limitations if these generation types are non-dispatchable. The methodologies behind the formulation of input data for the OPF model for each of these four categories may therefore be discussed in greater detail.

A.2 Bus Formulation

The division of Britain on a regional level is most suited to the level of resolution required by this analysis. This therefore divides the Island into the South East, the South West, London, the East, East Midlands, West Midlands, the North West, Yorkshire and the Humber, the North East, Wales and Scotland. Due to the small size of the North East, it was decided that this bus be merged with Yorkshire and the Humber as unnecessary detail can pose a threat to model convergence in practice. It should also be noted that this bus still has a smaller demand than both the South East and London.

For each of these regions the bus location was decided to be the main load location (largest city), in each region. While the global positioning system (GPS) location data of each region is not required as a bus input, these GPS coordinates will be further utilised later, in the Branch Formulation section.

Table 3: *Bus location specifications (regions, capitals and locations).*

Region	Largest City	GPS
South East	Brighton And Hove	50.8223711,-0.1373639
South West	Bristol	51.4545085,-2.5879675
London	London	51.5073321,-0.1278966
East	Norwich	52.6308914,1.2972594
East Midlands	Leicester	52.6365868,-1.1395656
West Midlands	Birmingham	52.4862263,-1.8905143
North West	Manchester	53.4807532,-2.2427672
North East, Yorkshire & The Humber	Leeds	53.8007312,-1.5492442
Wales	Cardiff	51.4815857,-3.1791789
Scotland	Glasgow	55.8642343,-4.2519078

Now that the buses have been determined, the properties for each of the 10 are required. The model requires the Bus (number), Type, Pd, Gd, Gs, Bs, area, Vm, Va, basekV, zone, Vmax and Vmin properties are required. Many of these, however, are not required for a model as generic as this, with all buses being in the same area, with simply a real load being specified. As such, besides configuration data such as the bus number and type (for which all buses will be PQ type besides the South East, which will be the slack bus), only the load and base voltage will have to be found. The initial voltage values inputted into the Newton-Raphson solver will simply be at a magnitude of 1 per unit (p.u.) and at a 0° angle.

For Britain the base voltage of the long-distance transmission system consists primarily of 400 kV and 275 kV overhead lines. This will be discussed in greater depth in the Branch Formulation section, but due to 400 kV lines being the most prevalent, 400 kV is used as the base voltage for the entire network. As for the loads themselves, these are taken from the most recently released spreadsheets from the United Kingdom Government (Dukes – Digest of United Kingdom Energy Statistics) [32]. The most recent year released at the time of writing is 2017. Given the branch data required is also available for 2017, this year was taken as the basis of the construction of the general model infrastructure (buses, branches and generators).

A.2.1 Bus Inputs

For each bus the following parameters must be defined:

- The Bus Number
- The Bus Type (all were PQ buses besides the slack bus)
- The power demanded P_{d} (load) MW
- The pre-iterative, initialised voltage magnitude $V_{\mathrm{m}} = 1\mathrm{V}$ p.u.

- The pre-iterative, initialised voltage angle $V_a = 0°$ p.u.
- The base voltage baseKV = 400 kV
- The minimum voltage level permitted V_{min} p.u.
- The maximum voltage level permitted V_{max} p.u.

A.2.2 Bus Specifications

The 10 busses are defined as follows:

Table 4: *The specified input values for each bus.*

Bus	Type	Pd	Vm	Va	basekV	Vmax	Vmin
1	3	4305.753425	1	0	400	1.05	0.95
2	1	2690.713899	1	0	400	1.05	0.95
3	1	4349.411502	1	0	400	1.05	0.95
4	1	2997.804	1	0	400	1.05	0.95
5	1	2354.482253	1	0	400	1.05	0.95
6	1	2730.946765	1	0	400	1.05	0.95
7	1	3493.016198	1	0	400	1.05	0.95
8	1	3876.75468	1	0	400	1.05	0.95
9	1	1696.432199	1	0	400	1.05	0.95
10	1	2756.56548	1	0	400	1.05	0.95

These specifications may be found in 'bus.txt' in 'DOI'. These inputs are consistent with PyPower / MatLab inputs with those utilised displayed above (A.2.1).

A.3 Branch Formulation

Branches connect the buses together, representing a simplified transmission system. In practice, the UK's long-distance electrical grid is a complex network of predominantly 400 kV and 275 kV power lines [12, 27].

While UK's National Grid Electricity System Operator publishes information pertaining to individual elements of this network, this information is far too detailed for scale of model this analysis seeks to utilise [25]. As such, while the information pertaining to these individual elements will be used to determine averaged branch specifications, the branch network itself ('to' and 'from' bus connections), will simply be constructed through visual inspection.

Furthermore, the distances of these lines will simply be based the distances between the geographic points selected for each region. While this is more generic, modelling an electrical transmission system at this scale using straight line connections is commonplace in the discussed literature. In practice this means that this model both simplifies loads down to single point in each region, and the branches between these points as straight lines. While certain distances may be over or underestimated, however, when a power flow analysis was conducted to test this model, losses were found to fall within the range of 1–3%, which is consistent with the UK's recorded transmission losses, though there is a lower net cable length [24, 26]. Further confirmation comes from the losses from different optimal power flow pricing scenarios, which calculated transmission losses in the same 2–3% range; though these losses were slightly higher, as would be expected due to the influence of economic dispatch analysis in the overall cost minimisation problem.

The specifics of the construction of this branch network are as follows. Firstly, only 400 kV and 275 kV lines were considered. Furthermore, as noted earlier in the bus formulation section, the grid, as it was in 2017 was considered. Through visual inspection the number of 400 kV and 275 kV lines connecting each region were determined.

Predominantly, but not exclusively this took the form of analysing the border between two regions. A notable exception, for example, was the London to Leicester (East Midlands) connection, which, given these regions are connected through East England, would have been routed through Norwich, a city further from each of two than they are from each other. This represents the only connection used between cities of non-neighbouring regions. Similar reasoning was applied, however, to the connections between neighbouring regions too, particularly when connections crossed closely between the border points of three regions (in which case the connections between the city nodes themselves were considered more closely), or in cases where the network briefly and temporarily diverged at a geographic border (to accommodate a greater residential or commercial demand area), but there otherwise a single line connecting the route between two regions' bus cities. From these determined connections, and the properties determined for each individual line, a single equivalent branch and its associated properties could be calculated.

Using the method above the model now consists of 10 buses, with branches connecting them. These branches are based on an inspection of regional connections, predominantly being border connections. These branches are furthermore represented as straight line connections between these regional city buses, consisting of a number of parallel 400 kV and 275 kV overhead lines (OHLs), for which an equivalent single line connection is calculated. In order to do so, however, properties must be determined for the 400 kV and 275 kV OHLs. The National Grid's individual asset data, as discussed earlier, can therefore be used to calculate averaged properties for these components [25].

From the data present in this table notes the specifications for 400 kV and 275 kV OHLs, cables and composite lines active in 2017 for Scottish Hydro Electric Transmission, Scottish Power Transmission, National Grid Electricity Transmission and Offshore Transmission. Of these, due to the prevalence of OHLs in long distance power transmission, only overhead line specifications will be considered. As a result, underground cables and composite lines will not have their specifications considered. The parameters of Offshore Transmission's branches, will also not be considered, due to their lines not being used for conventional overland purposes and thus being designed with different components; thus having differing properties to conventionally utilised OHLs.

For a given branch, calculated as an equivalent circuit of multiple parallel elements, the resistance, reactance, suseptance and capacity must be known and thus these same parameters must be calculated first for these component parallel lines. Extracting the data as described in the above paragraph a list of OHLs is determined for overland power transmission in the UK. This list, however, also consists of a number of short distance OHLs despite the desired branches to be modelled being representative of numerous long distance transmission lines. This distinction is important as long distance lines are typically designed to have lower resistance, reactance and suseptance as well as higher capacity. Thus, ranking the OHLs and taking those corresponding to those lines of greatest lengths, results in a more suitable list of OHLs. This list therefore excludes shorter lines (many of which are under one km in length, with some even having their line lengths simply noted as 0.000 km). The representative long distance lines then have their properties averaged to create a representative 400 kV and 275 kV line. The per km resistance, reactance and suseptance, as well as these capacities of these lines are then used to calculate the properties of the numerous branch connections. In total this process resulted in the specification of 14 branches connecting the 10 buses.

A.3.1 Branch Inputs

For each branch the following parameters must be defined:

- The first bus to which the branch connects
- The second bus to which the branch connects
- The resistance level of the branch: R p.u.
- The reactance level of the branch: X p.u.
- The susceptance level of the branch: B p.u.
- The maximum power flow level of the branch: Rate MVA

A.3.2 Branch Specifications

The 14 branches connecting the busses are specified as follows:

Table 5: *The specified input values for each branch.*

Bus1	Bus2	R	X	B	Rate
1	2	0.001953846	0.018883727	3.434808106	8400.792857
1	3	0.000603832	0.005835976	1.887146937	11201.05714
3	4	0.001674484	0.016183717	2.943696511	8400.792857
3	5	0.00151509	0.014643189	2.663486037	8400.792857
4	5	0.002607172	0.025198053	2.037038481	5600.528571
2	6	0.001970283	0.019042581	1.539423328	5600.528571
5	6	0.00070422	0.007295334	2.051409924	8211.17563
6	7	0.001195139	0.011550895	2.101021112	8400.792857
5	8	0.001048719	0.010135765	3.277545861	11201.05714
7	8	0.000653111	0.007130233	3.729915306	10821.82269
2	9	0.000925833	0.010107625	1.321856745	5410.911345
6	9	0.002257097	0.021814612	1.763517381	5600.528571
7	10	0.009348339	0.090350742	1.826013484	2800.264286
8	10	0.004556209	0.044035298	3.55986225	5600.528571

These values may also be found in 'branch.txt' in 'DOI'. These inputs are consistent with PyPower / MatLab inputs with those utilised displayed above (A.3.1).

A.4 Generator: Formulation

The final of the three components required to specify the input parameters for a power flow (PF) analysis are the generators, of which there are 275 instances in total. As was discussed earlier in the bus formulation section, given that the most recent regional load data is from 2017, so too was the branch and generator taken from this same year. Power station data is released on an annual basis by the UK's Department for Business, Energy & Industrial Strategy and will be used for the basis of constructing the generator list [11].

From the data there is a list all generators active in 2017 of over a 30 MW capacity, with other generation being noted on a national level. Additionally, there is a second list of all generators active, including some, but not all, of those of a smaller size. The smallest size, for example, is 100 kW, but individual photovoltaic setups, for example, are not noted. The sum of all individual plants in the second list, however, is still of a significantly lower capacity than the summed capacities of the 2017 list inclusive of the aggregate small generators.

As such, after factoring out Northern Ireland (as only the island of Britain is considered), the aggregate small generator output from the first list, was proportionally allocated to each region by the capacities of each generation type (*e.g.* wind, solar, hydro, *etc.*. . .) present in the more regionally extensive, but incomplete, second list. Thus the final plant list consists of all individual plants noted in from the first list, as well as the aggregate generation from the first list, divided up between the regions in proportion to the capacity of the generator types present within them as per the second list. As mentioned above, this resultant list consists of 275 generators, which in turn are associated with each of the 10 buses regardless of if they are an individual generator or a regionally aggregated ‘other’ generator.

A.4.1 Generation: Dispatchable vs Non-Dispatchable Sources

Now that the list of generators is completed, their specifications must also be determined. Specifically, their capacities must be known. In the OPF model, their capacity, however, is presumed to be their maximum dispatchable capacity, subject to the given cost function. As such, for example, if a wind farm of a 100 MW capacity were generating at half that amount (50 MW), then its capacity as far as this model is concerned, is 50 MW, as that is the maximum amount of power which this generator could dispatch for this instance of time.

This presents a distinction between dispatchable (*e.g.* photo-voltaic (PV), wind) and non-dispatchable (nuclear, gas, coal) power sources. While an economic dispatch model may choose to utilise a dispatchable power source at whatever share of its maximum capacity is required, this is not the case for non-dispatchable sources. Thus, dispatchable power sources will have their generation information and cost functions inputted in accordance with the standard requirements of OPF modelling, while for non-dispatchable sources, the effective maximum power input must be determined and used in lieu of the generator’s rated capacity.

Although the economic dispatch component of the OPF analysis will vary the power output for dispatchable power sources, for non-dispatchable sources, aggregate data of the UK’s power output composition will be used to determine the output of each non-dispatchable power source. This output will be proportionally divided up between all generators of a given type as per the generator’s capacities.

A.4.2 Generator Inputs

For each generator the following parameters must be defined:

- The bus to which the generator is connected
- The initialised real power output of the generator P_{g} MW
- The initialised reactive power output of the generator Q_{g} MVAR
- The maximum reactive power level Q_{max} MVAR
- The minimum reactive power level Q_{min} MVAR
- The MVA base of the generator (100 MVA)
- The maximum real power output of the generator P_{max} MW
- The minimum real power output of the generator P_{min} MW

A.4.3 Generator Specifications

The 275 generators may be found with their specifications may be found in ‘gen.txt’ in ’DOI’. These inputs are consistent with PyPower / MatLab inputs with those utilised displayed above (A.4.2).

A.5 Cost Function: Formulation

In order to perform a simultaneous economic dispatch analysis with a power flow analysis and thus perform an optimal power flow analysis, cost data is required for every generator in the system. These generator cost functions are calculated for each generator type. This 'cost', though often economic in practice, could simply be any variable the model seeks to minimise and as such could represent environmental costs also, or both economic and environmental costs via the implementation of an environmental pricing mechanism, such as a carbon tax.

PyPower supports polynomial cost function modelling. These functions may be used to model both the fixed and variable costs of power plant operation. The REPA research group proposes generic generation costs for a number of generator types as well as ramp rate and emissions data [33]. In this paper's second table, the fixed and variable costs (operation and maintenance (O&M)) are listed, which, excluding additional considerations and policies such as subsidies, tariffs and carbon tax, would result in marginal generation costs (levelized cost of energy (LCOE)) as per the following equation:

$$C_{\text{total}} = C_{\text{fixed}} + P_G(C_{\text{variable}} + C_{\text{fuel}}), \tag{A.1}$$

Where, for a given generator, C_{total} is total cost of operation of a power plant (in GBP/h), C_{fixed} is the fixed operating and maintenance cost of the power plant (in GBP/h), P_G is the power output of the plant for the given hour (in MW), C_{variable} is the variable operating and maintenance cost of the power plant (in GBP/MWh) and C_{fuel} is the fuel cost (in GBP/MWh).

Thus it can be seen that the cost function is a linear function, which is compatible with PyPower's requirements. The first variable, Fixed O&M (operation and maintenance) is a constant, as determined by multiplying the capacity of the power plant by its hourly cost per MW. 'Power Generated' (MWh) is simply the power outputted by the generator, which is to be calculated by the optimal power flow model itself. 'Variable O&M' and 'Fuel Cost' are both to be taken as inputs which scale with power generated and are both to be measured in price / MWh.

While the REPA data will be used for pollution figures, for the above equation it would be preferable to have data specific to the United Kingdom. Leigh Fisher published a revision of a 2013 report from the UK's Department for Business, Energy & Industrial Strategy where various costs for differing generator types are listed [22]. This same department also, and most recently, published another report on Electricity Generation Costs in 2016 [10]. Thus, of the three sources noted in this section the 2016 government paper will be used first and foremost, with the Leigh Fisher paper and REPA papers will be used in the event that the government report has insufficient information. It should be noted that the government report was sufficient in providing adequate information on this subject for the base cost functions, but as it does not note carbon outputs, the other sources must be considered to specify carbon tax costs.

A.5.1 Cost Function: Carbon Tax

For the purposes of this paper emissions reduction will specifically focus on CO_2 production from generators. Carbon emissions vary greatly between sources and as such, tCO_2/MWh must be determined for each generator type. These carbon outputs will then in turn be multiplied by a given carbon tax to determine a cost/MWh. Prices determined may then be integrated into the cost functions for each generator type, thus creating new cost functions inclusive of carbon pricing.

The cost function equation may be modified to include this consideration:

$$C_{\text{total}} = C_{\text{fixed}} + P_G(C_{\text{variable}} + C_{\text{fuel}} + P_{CO_2} \times T_{CO_2}), \tag{A.2}$$

Where, for a given generator, C_{total} is total cost of operation of a power plant (in GBP/h), C_{fixed} is the fixed operating and maintenance cost of the power plant (in GBP/h), P_G is the power output of the plant for the given hour (in MW), C_{variable} is the variable operating and maintenance cost of the power plant (in GBP/MWh), C_{fuel} is the fuel cost (in GBP/MWh), P_{CO_2} is the generator's CO_2 pollution output (in tonnes of CO_2 per MWh: tCO_2/MWh) and T_{CO_2} is the carbon tax rate (in GBP/tCO_2).

In this equation, two variables are included to note the carbon tax, these being the 'Carbon Output' (tCO_2/MWh) and 'Carbon Tax' (GBP/tCO_2). As noted in the previous section, the most recent government source has insufficient information to provide values for the carbon outputs, however, the REPA paper specifically provides such values. The carbon tax rate of the UK may be taken from present or expected future values, or hypothetical values and will also be the same for all generator types, therefore negating a requirement to source generator type specific values for this input.

While bus, branch and generator inputs all have unique input values, this is not the case for the cost functions. Buses, for example, utilise geographic consumption data provided by the UK government, while branches have unique distances used to scale their 400 kV and 275 kV specifications. Generators are the most specific of all these cases, as they are individually specified; either on a per generator basis (the majority of cases), or a unique aggregate equivalent for very small generators.

A.5.2 Cost Inputs

As noted in equation A.2, the the carbon tax inclusive cost function can be modelled, where:

- C_{total} is total cost of operation of a power plant (in GBP/h),
- C_{fixed} is the fixed operating and maintenance cost of the power plant (in GBP/h),
- P_G is the power output of the plant for the given hour (in MW),
- C_{variable} is the variable operating and maintenance cost of the power plant (in GBP/MWh),
- C_{fuel} is the fuel cost (in GBP/MWh),
- P_{CO_2} is the generator's CO_2 pollution output (in tonnes of CO_2 per MWh: tCO_2/MWh) and

- T_{CO_2} is the carbon tax rate (in GBP/tCO_2).

A.5.3 Cost Specifications

The costs of the 275 generators may be found with their specifications may be found in 'genCost.txt' in 'DOI'. These inputs are consistent with PyPower / MatLab inputs with those utilised displayed above (A.5.2).

B Appendix: Regional Loads and (Polluting and Total including Non-Polluting) Capacities

Figure 13: *Regional Demand (MW).*

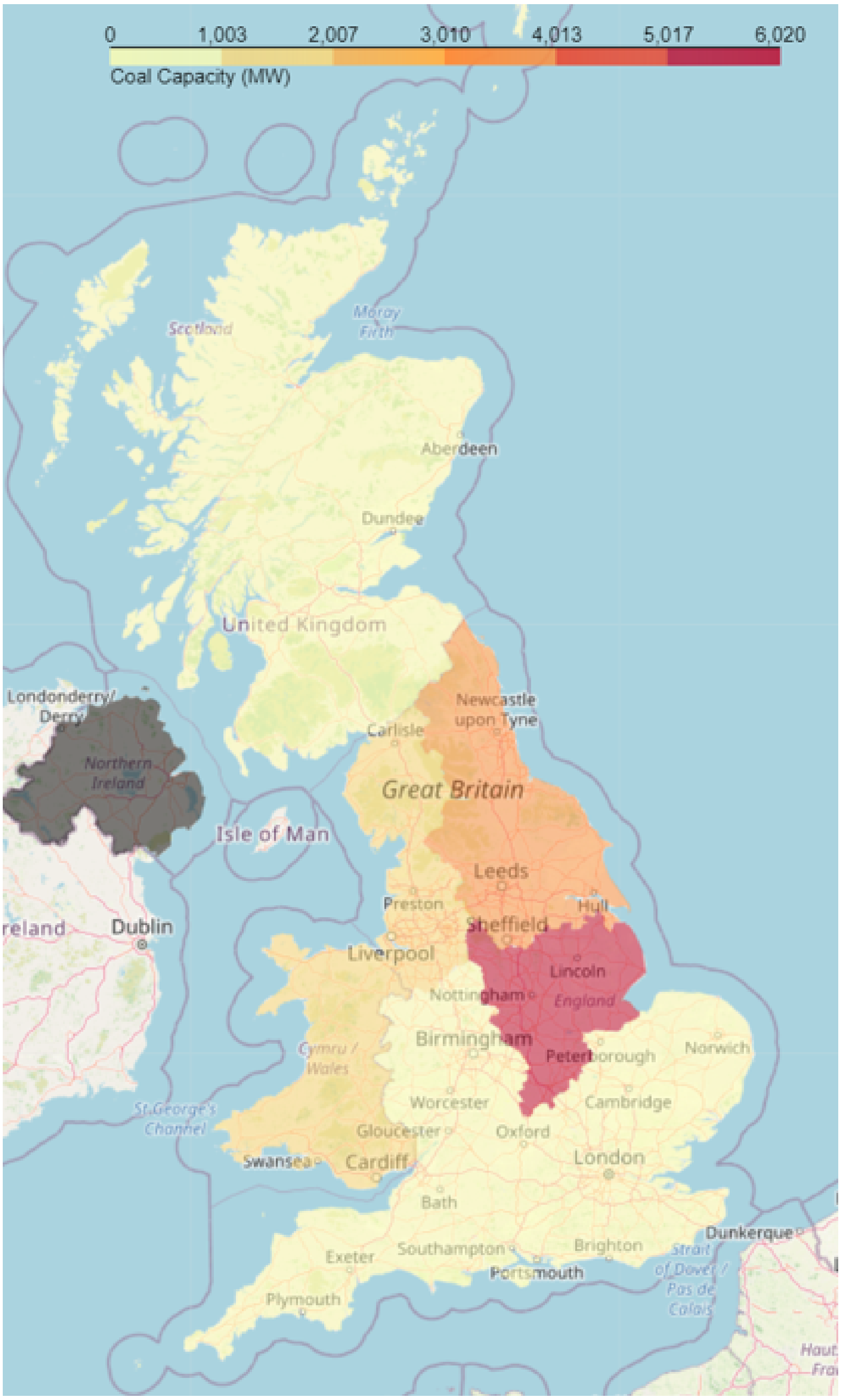

Figure 14: *Regional Coal Capacity (MW).*

Figure 15: *Regional CCGT Capacity (MW).*

Figure 16: *Total Regional Capacity (MW).*

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